Nuclear Structure Functions from Constituent Quark Model

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Abstract

We have used the notion of the constituent quark model of nucleon, where a constituent quark carries its own internal structure, and applied it to determine nuclear structure functions ratios. It is found that the description of experimental data require the inclusion of strong shadowing effect for $x < 0.01$. Using the idea of vector meson dominance model and other ingredients this effect is calculated in the context of the constituent quark model. It is rather striking that the constituent quark model, used here, gives a good account of the data for a wide range of atomic mass number from $A = 4$ to $A = 204$.

I. INTRODUCTION

The measurement of the nuclear structure function, $F_2^A(x, Q^2)$, in Deep Inelastic Scattering (DIS) indicates that the structure function of a bound nucleon is different from a free nucleon. Very precise data on the proton structure function from HERA [1] [2] [3] [4] and NMC [5] which cover a wide range of kinematics, both in x and Q^2, sets a stringent demand on the understanding of nucleon structure function in the nuclear environment. The difference between the parton distributions in the free and the bound nucleon, that is, $f_{\frac{i}{A}}(x, Q^2) \neq f_{\frac{i}{p}}(x, Q^2)$ is attributed to the nuclear effects. These effects are characterized in the structure function ratio

$$R = \frac{F_2^A(x, Q^2)}{F_2^D(x, Q^2)} \tag{1}$$

There are a host of interpretations that describe the DIS data on the nuclear targets pioneered by the European Muon Collaboration (EMC effect) and it is believed that various mechanisms are responsible for the behavior of R in various kinematical intervals:

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i)The Shadowing effect at $x < 0.1$ where $R \leq 1$.

ii)Anti-shadowing effect at $0.1 \leq x \leq 0.3$ where $R \geq 1$.

iii)The EMC effect for $0.3 \leq x \leq 0.7$ where $R \leq 1$.

iv)The Fermi motion for $x > 0.7$ where $R \geq 1$.

For a detailed study of these mechanisms and other theoretical models sea Arneode [6]. Currently, there are fairly accurate data on the structure function ratio $R = \frac{F_2^A(x,Q^2)}{F_2^D(x,Q^2)}$ that has been obtained by different experimental groups [7] for a variety of nuclei ranging in mass number from $A = 4$ to $A = 208$. These data, now, permit us to investigate the x and Q^2 dependence of the $F_2^A(x, Q^2)$. One feature of the data, almost universal, is the shape of the x-dependence of the F_2^A and is basically the same for every nuclei, thus making it rather easy to fit in a wide range of models with totally different underlying assumptions. Our interest here is to find a description for the nucleon structure which can be extended to the nuclei; yet maintaining some universal features common to both. To this end, our starting point is the study of the internal structure of the constituent quark which is common to both nucleon and nuclei. Its internal structure is universal and governed by the annihilation and pair production in Quantum Chromodynamics. We assert that at high enough Q^2, it is the structure of this constituent quark that is being probed, whereas, at low Q^2 it behaves as a structure-less valence quark. Such a model has been applied successfully to the proton structure for the entire range of kinematics in x and Q^2 [8]. Encouraged by this results, we think that the nuclear environment can only alter the distribution of the constituent quark, leaving its internal structure intact. The fact that valence quark distribution in nuclei receives a distortion as opposed to its distribution in a free nucleon is very similar to the nucleon swelling and the relaxation of the confinement of the constituent quark within the boundaries of a nucleon embedded in a nucleus. To put it differently, a nucleon in a nucleus will relax its boundaries and will occupy broader space, preserving the universality of the constituent quark of the relaxed nucleon. None of the number of the constituent quarks, its internal structure, and the average momentum of the constituent will change. Thus, here we will be dealing with a collection of structure-full U, D-type constituent quarks participating in DIS.

II. FORMALISM

To proceed quantitatively, we need to employ a model which is compatible with the deep inelastic scattering data on a free nucleon and then we impose the restrictions due to nuclear effects for the bound nucleons.The picture that comes in mind is the so-called *Valon* model of R.C.Hwa [9]. In this model the nucleon is considered as a system of bound constituent quarks, which themselves have structure. The bound state problem is a nonperturbative effect which is taken into account in the distribution of constituent quarks. The structure of constituent quark is produced perturbatively and is free of bound state problem. In such a picture the structure function of a free nucleon is the convolution of constituent quark distribution in a nucleon and the structure function of the constituent quark itself:

$$F_2^N(x, Q^2) = 2e_U^2 \int dy G_{\frac{U}{N}}(y) f^c(x, Q^2) + e_D^2 \int dy G_{\frac{D}{N}}(y) f^c(x, Q^2) \tag{2}$$

where $G_{\frac{U}{N}}(y)$ and $G_{\frac{D}{N}}(y)$ are the distribution of U and D-type constituents in a nucleon and $f_2^C(z, Q^2)$ is the structure function of the constituent calculated in QCD. At sufficiently high Q^2, $f_2^C(z, Q^2)$ can be described accurately in the Leading-Order results in QCD. They are also calculated in the Next-to-Leading Order[8]; but here we will restrict ourselves to the leading order, because, we are mainly interested in the application of the model and it describes the nuclear structure functions sufficiently accurately. The moments of $f_2^C(z, Q^2)$ can be expressed in terms of the evolution parameter:

$$s = ln\frac{ln\frac{Q^2}{\Lambda^2}}{ln\frac{Q_0^2}{\Lambda^2}} \tag{3}$$

where $Q_0 = 0.233\frac{GeV^2}{c}$ and $\Lambda = 0.345\frac{GeV^2}{c}$ are scale parameters determined from the data. The moments of the singlet and non-singlet constituent quark structure function in the leading order solution of the renormalization group equation are given as[9]:

$$
\begin{aligned}
M^{NS}(n, Q^2) &= \exp(-d_{NS}\ s), \\
M^S(n, Q^2) &= \tfrac{1}{2}(1+\rho)\exp(-d_+\ s) + \tfrac{1}{2}(1-\rho)\exp(-d_-\ s),
\end{aligned}
\tag{4}
$$

The anomalous dimensions, d'^s, and other associated parameters are :

$$
\begin{aligned}
\rho &= (d_{NS} - d_{gg})/\Delta, \\
\Delta = d_+ - d_- &= [(d_{NS} - d_{gg})^2 + 4d_{gQ}d_{Qg}]^{1/2}, \\
d_{NS} &= \tfrac{1}{3\pi b}[1 - \tfrac{2}{n(n+1)} + 4\sum_2^n \tfrac{1}{j}], \\
d_{gQ} &= \tfrac{-2}{3\pi b}\tfrac{2+n+n^2}{n(n^2-1)}, \\
d_{Qg} &= \tfrac{-f}{2\pi b}\tfrac{2+n+n^2}{n(n+1)(n+2)}, \\
d_{gg} &= \tfrac{-3}{\pi b}[-\tfrac{1}{12} + \tfrac{1}{n(n-1)} + \tfrac{1}{(n+1)(n+2)} - \tfrac{f}{18} - \sum_2^n \tfrac{1}{j}], \\
d_\pm &= \tfrac{1}{2}[d_{NS} + d_{gg} \pm \Delta], \\
b &= (33 - 2f)/12\pi,
\end{aligned}
\tag{5}
$$

Since these moments, $M(n, s)$, are known, using inverse Mellin transformation technique, we can obtain the distributions of various components in the constituent quark; namely, valence, sea quarks, and gluon distributions, $P^{v(sea,g)}$. They are as follows:

$$
\begin{aligned}
P^{v(s)g}(z, Q^2) &= \tfrac{1}{2\pi i}\int_c dn z^{-n+1} M_{v(s)g}(n, s), \\
M_v(n, s) &= M^{NS}(n, s), \\
M_s(n, s) &= (2f)^{-1}(M^S(n, s) - M^{NS}(n, s)), \\
M_g(n, s) &= M_{gQ}(n, s),
\end{aligned}
\tag{6}
$$

and $M_{gQ}(n, s)$ is the quark-to-gluon evolution function given by:

$$M_{gQ}(n, S) = \Delta^{-1}d_{gQ}[\exp(-d_+ s) - \exp(-d_- s)], \tag{7}$$

To account for the $SU(2)$ asymmetry of the nucleon sea which is evident form the violation of the Gottfried sum rule, we follow the same procedure as in [7] and replace the distribution of d-quark in the sea by:

$$P_{d/C}(z, Q^2) = \frac{1}{(1-x)} P_{u/C}(z, Q^2), \tag{8}$$

The calculation of the moments given in eq.(4) is simple. Instead of exhibiting the moment distribution, we present the results in parametric form. That is, for every s we fit the moments by a form give below:

$$\begin{aligned}
P_C^v(z, Q^2) &= a_v(Q^2) \, z^{b_v(Q^2)} \, (1-z)^{c_v(Q^2)}, \\
P_C^s(z, Q^2) &= \sum_1^2 a_{s_i}(Q^2)(1-z)^{b_{s_i}(Q^2)}, \\
P_C^g(z, Q^2) &= \sum_1^2 a_{g_i}(Q^2)(1-z)^{b_{g_i}(Q^2)}
\end{aligned} \tag{9}$$

for the valence, sea quarks, and gluon respectively. The dependence of the coefficients a_j, b_j and c_j on Q^2, or rather on s is given in the appendix. To complete the calculation of F_2^N, we also need to specify constituent quark distributions, $G_{\frac{U}{N}}$ and $G_{\frac{D}{N}}$. The simplest approach to evaluate these distributions is to write down the inclusive momentum distribution of the constituent quark in a free nucleon. We will limit ourselves to proton here. these inclusive distributions can be written as:

$$G_{UUD}(y_1, y_2, y_3) = \alpha \, y_1^a \, y_2^a \, y_3^b \, \delta(y_1 + y_2 + y_3 - 1) \tag{10}$$

where $a = 0.65$ and $b = 0.35$ are two free parameters and y_1, y_2 refer to U-type and y_3 refers to the D-type constituents. By double integration over the unspecified variable we get the exclusive distributions:

$$\begin{aligned}
G_{\frac{U}{P}}(y) &= [B(a+1, a+b+2)]^{-1} \, y^a \, (1-y)^{a+b+1} \\
G_{\frac{D}{P}}(y) &= [B(b+1, 2a+2)]^{-1} y^b \, (1-y)^{2a+1}
\end{aligned} \tag{11}$$

where $B(x, y)$ is the Euler Beta function. The above distributions satisfy also the following normalization conditions:

$$\int_0^1 dy \, G_{\frac{U}{P}}(y) = \int_0^1 dy \, G_{\frac{D}{P}}(y) = 1 \tag{12}$$

In figure 1, we present the result for $F_2^P(x, Q^2)$ for several values of x and Q^2, its agreement with the experimental data in a wide range of kinematics is rather good.

Having determined the nucleon structure function, next we extend the same procedure to the nuclear medium. As we mentioned earlier, in the nuclear medium the constituent quark distribution has to be modified, but its structure function, or rather parton distribution in a constituent will not be affected by the nuclear environment. The modification to $G_{\frac{C}{P}}(y)$ is achieved by introduction of a distortion factor, δ. Hence, we write the inclusive momentum distribution as follows :

$$G'_{UUD}(y) = \alpha(y_1 y_2)^{a+\delta} y_3^{b+\delta} \delta(y_1 + y_2 + y_3 - 1) \tag{13}$$

which leads to :

$$\begin{aligned}
G'_{\frac{U}{P}}(y) &= \alpha \, y^{a+\delta} \, (1-y)^{a+b+2\delta+1} \\
G'_{\frac{D}{P}}(y) &= \alpha' \, y^{b+\delta} \, (1-y)^{2a+2\delta+1}
\end{aligned} \tag{14}$$

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with :

$$\alpha = [Beta(a + \delta + 1, a + b + 2\delta + 2)]^{-1}$$
$$\alpha' = [Beta(b + \delta + 1, 2a + 2\delta + 2)]^{-1} \tag{15}$$

we note that δ is a function of atomic number, A. In Ref. [8] a probabilistic argument is used to find δ. Here we do generalize the same result as follows :

$$\delta = \frac{0.0104 - 0.014 \ln A + 0.014 (\ln A)^2}{1 - 0.83 \ln A + 0.295 (\ln A)^2 - 0.024 (\ln A)^3} \tag{16}$$

So far we have determined the constituent quark distribution in a bound nucleon. We need to find its distribution in a nucleus. For that matter, however, one needs also to know the nucleon distribution in the nucleus. we can use the Fermi distribution and arrive at the following result :

$$\phi_N^A(z) = \int_{k_F}^{|k|} d^3 k \; \rho(k) \; \delta(z - 1 - \frac{k^+}{M_n}) \tag{17}$$

where $k^+ = k^0 + k^z$ and $k^0 = M_N$ are the light cone coordinates and $\rho(k) = \frac{3}{4\pi k_F^3}\theta(k_F - |k|)$ is the nucleon density in the nucleus. So, straightforward integration would yield the nucleon distribution in a nucleus. However, this also means that the bound nucleon is nearly on shell and therefore, the nuclear binding effect is neglected. To be more realistic, and include this effect, we replace $z - 1$ by $z - \eta_A$ where $\eta_A = 1 - \frac{B_A}{M_N^*}$ and $M_N^* = M_N - B_A$ is the effective mass of the nucleon. With these fine tunings we arrive at the following function for the nucleon distribution in a nucleus :

$$\phi_N^A(z) = \frac{3}{4}\frac{M_N^3}{k_F^3}(\frac{k_F^2}{M_N^2} - (z - \eta_A)^2) \; ; \eta_A - \frac{k_F^A}{M_N}\langle \; z \; \langle \eta_A + \frac{k_F^A}{M_N}$$
$$\phi_N^A(z) = 0 \; ; otherwise \tag{18}$$

The values of the Fermi momentum, k_F^A, and the binding energy, B_A, are given by the requirements of nuclear physics. Going one step further, we carry the Fermi motion to the constituent quark level. This is done easily by renormalization of the distribution function give in Eq.(14) this leads to the following constituent quark distribution per proton in a nucleus :

$$G_{\frac{U}{P}}^A(y) \approx (\frac{1}{\eta})G'_{\frac{U}{P}}(\frac{y}{\eta}) + \frac{1}{10}\lambda^2(\frac{d^2}{dz^2})(\frac{1}{z})G'_{\frac{U}{P}}(\frac{y}{z}) \mid_{z=\eta} \tag{19}$$

$$G_{\frac{D}{P}}^A(y) \approx (\frac{1}{\eta})G'_{\frac{D}{P}}(\frac{y}{\eta}) + \frac{1}{10}\lambda^2(\frac{d^2}{dz^2})(\frac{1}{z})G'_{\frac{D}{P}}(\frac{y}{z}) \mid_{z=\eta} \tag{20}$$

where $\lambda = \frac{k_F}{M_N^*}$. Having collected all the ingredients, now we are in a position to evaluate the nuclear structure function ratios, $R = \frac{F_2^A(x,Q^2)}{F_2^D(x,Q^2)}$. Notice that all of our discussions about the constituent quark distribution are pertinent to proton. Finally, taking the neutron excess in the nucleus into consideration, we get :

$$F_2^A(x, Q^2) = (\frac{1}{A})\{F_{2A}(x, Q^2) - \frac{1}{2}(N - Z)[F_2^n(x, Q^2) - F_2^p(x, Q^2)]\} \tag{21}$$

where $F_2^p(x, Q^2)$ and $F_2^n(x, Q^2)$ are the free proton and free neutron structure functions, respectively, and $F_{2A}(x, Q^2)$ is obtained using Eq.(1). The results of the model is presented in figure 2 for a variety of nuclei ranging from $A = 4$ to $A = 204$. As it is apparent from Fig.(2), we see that the model calculation is reasonable only down to $x \geq 10^{-1}$. For smaller values of x, $x \leq 10^{-1}$ an extra ingredient is required. An obvious and natural place to look for the dynamics of small x would be the shadowing effects, where the growth of parton densities are suppressed due to recombination and annihilation of the partons. There are some attempts in the literature to calculate the shadowing corrections, pioneered by Qiu [12]. However we found that such a parameterization is inadequate in describing the data. Following Ref.[12] we modify the shadowing correction of Qiu in two ways: (1) we take the general idea of vector meson dominance which describes low Q^2 virtual photon interaction, to model the approximate scaling of shadowing by pomeron exchange. It turns out that the experimental data on $F_{2p}(x, Q^2)$ up to $x < 0.05$ can be well described by a sub-asymptotic pomeron with the effective anomalous dimension going down logarithmically with x [13].(2) The magnitude of the shadowing effect in such models is sensitive to the value of vector meson-nucleon cross section. This sensitivity is explored by G. Show [14] and it is described by a quantity :

$$\frac{A_{eff}}{A} = \frac{\sigma_{\gamma\ A}}{A\sigma_{\gamma\ N}} \tag{22}$$

This ration is particularly manifested for heavy nuclei and reflects the ratio of nuclear radius R to the mean free path, λ, of the vector meson inside the nucleus; which also depends on the nuclear density . That is, the shadowing effect at small x values increases with increasing nuclear radius and density. Utilizing these modifications to the original work of Qiu [12], finally we present the following modification of parton distributions in a nucleus :

$$\begin{aligned} P_{C/A}^s(z, Q^2) &= R_s(x, Q^2, A_{eff})P_{C/N}^s(z, Q^2), \\ P_{C/A}^v(z, Q^2) &= P_{C/N}^v(z, Q^2), \end{aligned} \tag{23}$$

where superscript $s(v)$ stands for sea (valence) components and $R_s(x, Q^2, A_{eff})$ is the shadowing factor affecting parton distributions of the constituent quark. These distributions in the nuclear medium are compared to the corresponding distribution in a free nucleon. The parameterized form of this factor is :

$$R_s(x, Q^2, A_{eff}) = R_s(Q^2, A_{eff})(0.94 + 0.076A_{eff}^{0.5})x^{0.5}ln(x)) \tag{24}$$

And A_{eff} is given by:

$$A_{eff} = -10.97 - 0.704A + .0042A^2 - 0.00019A^{2.5} + 8.63A^{0.5} \tag{25}$$

In equation (24) $R_s(Q^2, A_{eff})$ is evaluated by the nth-moment equations for the parton distribution in the sea of a constituent quark [15]:

$$\begin{aligned} < F_{c/N}^s(Q^2) >_n =&< F_{c/N}^s(Q_c^2) >_n K_{qq}^n(Q^2) + < F_{c/N}^g(Q_c^2) >_n K_{qg}^n(Q^2) + \\ &< F_{c/N}^v(Q_c^2) >_n K_{NS}^n(Q^2) \end{aligned} \tag{26}$$

where $< F(Q^2) >_n = \int_0^1 z^{n-2} P(z, Q^2) dz$. Obviously, nuclear shadowing do not affect the distribution of valence quarks in the constituent quarks. If the momentum loss owing to the recombination process of the shadowed sea quarks is negligible, the momenta carried by each parton component would be conserved separately and approximately,

$$< F^s_{c/N}(Q^2) >_2 \cong < F^s_{c/A}(Q^2) >_2$$
$$< F^g_{c/N}(Q^2) >_2 \cong < F^g_{c/A}(Q^2) >_2 \tag{27}$$

and the following inequalities will be satisfied for $Q^2 > Q_0^2$[15] :

$$< F^{s(g)}_{c/N(A)}(Q^2) >_2 \gg < F^{s(g)}_{c/N(A)}(Q^2) >_3 \tag{28}$$

As a result, following Re. [12] we acquire:

$$R_sea(Q_0^2, A_{eff}) = 1 - k_s(Q_0^2)(A_{eff}^{1/3} - 1) = \lim_{x \to 0} \frac{F^s_A(x, Q^2)}{F^s_N(x, Q_0^2)} \approx \frac{< F^s_{C/N}(Q_0^2) >_3}{< F^s_{C/A}(Q_0^2) >_3} \tag{29}$$

$$R_g(Q_0^2, A_{eff}) = 1 - k_g(Q_0^2)(A_{eff}^{1/3} - 1) = \lim_{x \to 0} \frac{F^g_A(x, Q_0^2)}{F^g_N(x, Q_0^2)} \approx \frac{< F^g_{C/N}(Q_0^2) >_3}{< F^g_{C/A}(Q_0^2) >_3} \tag{30}$$

where $K_{s(q)}$ is the shadowing strength and and depends on the starting scale Q_0^2 for the evolution to set in. It is easy to verify that the primitive shadowing for gluon in a constituent quark is weaker than that for the sea partons[12] at such scale. The form of shadowing correction is in terms of evolution kernels is given as:

$$R_s(Q^2, A_{eff}) = R_s(Q_0^2, A_{eff}) \frac{< F^s_{c/N}(Q_0^2) >_3 K^3_{qq}(Q^2) + < F^g_{C/N}(Q_0^2) >_3 K^3_{qg}(Q^2) + < F^v_{C/N}(Q_0^2) >_3 K^3_{NS}(Q^2)}{< F^s_{C/N}(Q_0^2) >_3 K^3_{qq}(Q^2) + \lambda < F^g_{C/N}(Q_0^2) >_3 K^3_{qg}(Q^2) + R_s(Q_0^2) < F^v_{C/N}(Q_0^2) >_3 K^3_{NS}(Q^2)} \tag{31}$$

where K^n_{ab} are the evolution kernels. Defining $s = lnL$, these kernel functions are[15]:

$$K^n_{qq}(Q^2) = \alpha_n L^{-a_n^-} + (1 - \alpha_n) L^{-a_n^+}$$
$$K^n_{qg} = \beta_n (L^{-a_n^-} - L^{-a_n^+})$$
$$K^n_{NS}(Q^2) = \left[\alpha_n L^{-a_n^-} + (1 - \alpha_n) L^{-a_n^+} - L^{-a_{NS}^n} \right]$$
$$a_{NS} = \frac{\gamma_{qq}}{\beta_\circ} , a_\pm^n = \frac{\gamma^{\pm n}}{\beta_\circ}$$
$$\alpha_n = \frac{\gamma_{qq} - \gamma^+}{\gamma^- - \gamma^+}$$
$$\beta_n = \frac{\gamma_{qq}}{\gamma^- - \gamma^+} \tag{32}$$

III. CONCLUSION

We have calculated the nucleus structure function ratio in the context of constituent quark picture, utilizing the essence of the so called valon model. As we can see from the figures; the model describes the data rather well for a wide range of A. It appears that

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the Q^2 dependence of the ratio of structure functions is week. The shadowing effect is quite large for small value of x, ($x\langle 0.01$) and dies away very rapidly as x increases. Also, the shadowing correction becomes more pronounced with increasing atomic number A. We further note that in this model, which is based on the constituent quark structure, we have not included explicitly the anti-shadowing effects notwithstanding that there are some debates on the subject particularly pertinent to the middle range of x. we simply did not need it to describe the data. Our anticipation is that such an effect would not be very large .

IV. APPENDIX

In this appendix we provide numerical values for the coefficients given in equation ( ). These are the results of our Q^2 dependence parameterization of parton distributions in a constituent quark of proton. They are calculated in the leading order.

$$
\begin{aligned}
a_{s1} &= -0.119 + 0.0296\exp(s/0.66) \\
b_{s1} &= 5.510 + 8.50\exp(s/0.95) \\
a_{s2} &= -0.080 + 0.066\exp(s/3.12) \\
b_{s2} &= -1.194 + 2.303\exp(s/2.76) \\
a_{g1} &= -2.490 + 1.027\exp(s/0.69) \\
b_{g1} &= 5.920 + 6.440\exp(s/0.80) \\
a_{g2} &= .174 + .345s^{0.09} \\
b_{g2} &= -1.407 + 1.763\exp[s/2.42]; \\
a_v &= -0.300 + 1.254s - 0.368s^2 + 0.034s^3 \\
b_v &= 0.196 + 1.286\exp[-s/3.14] \\
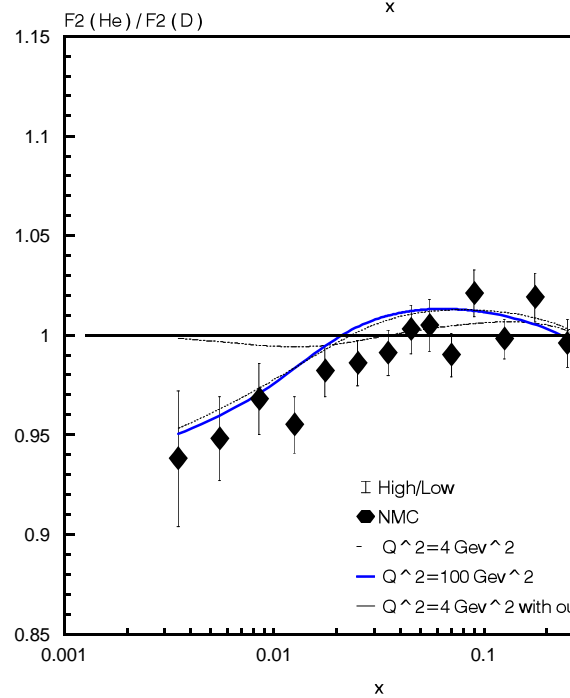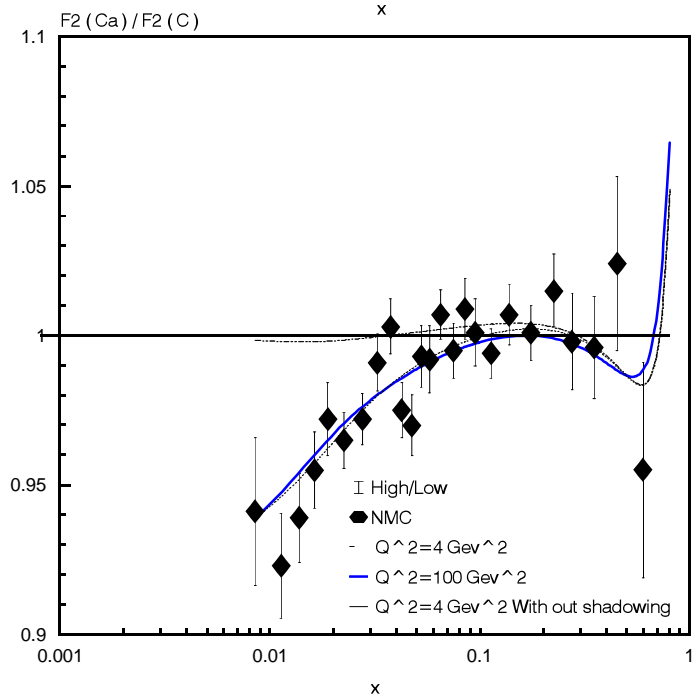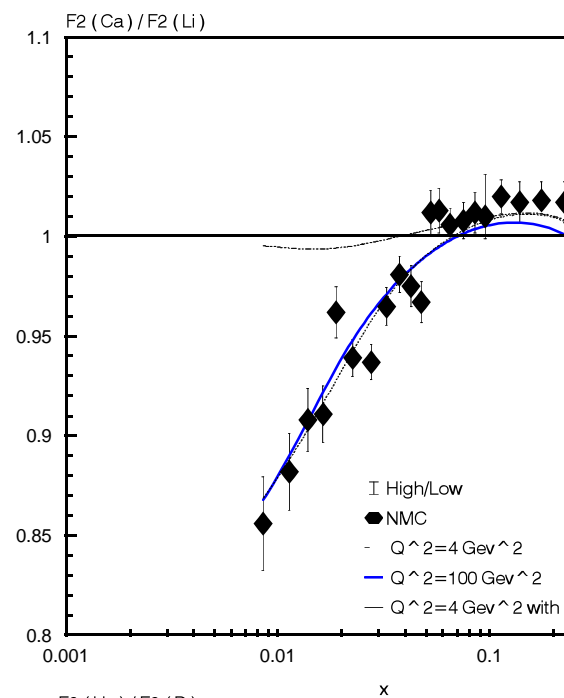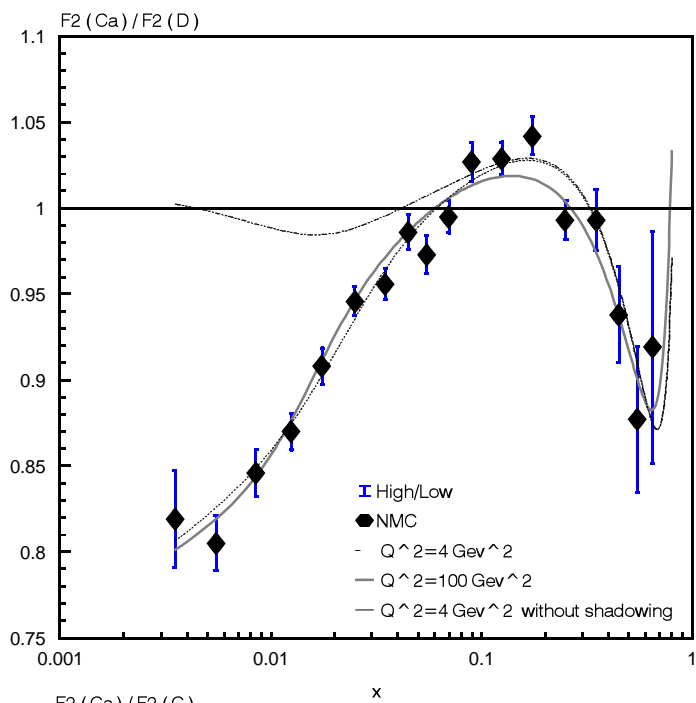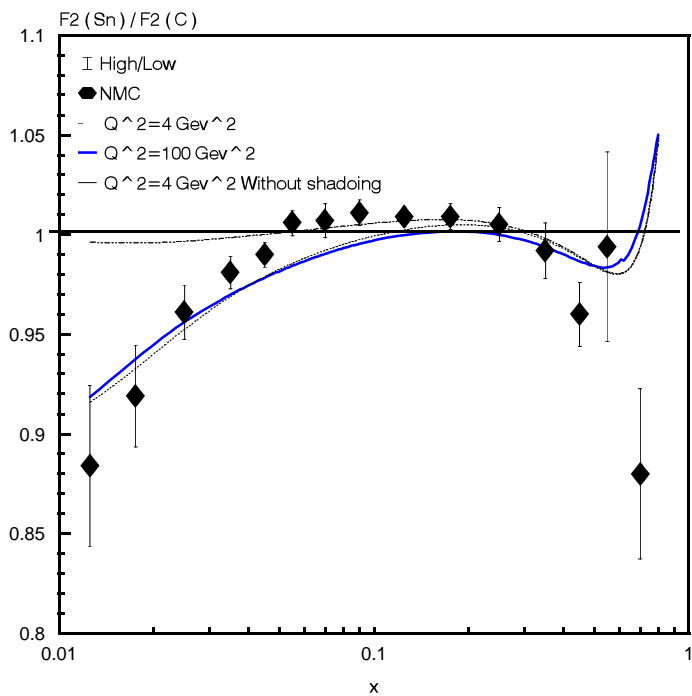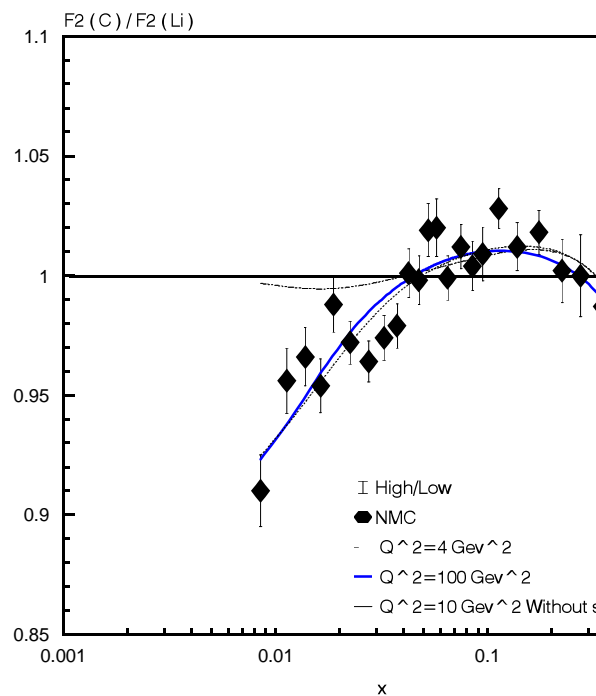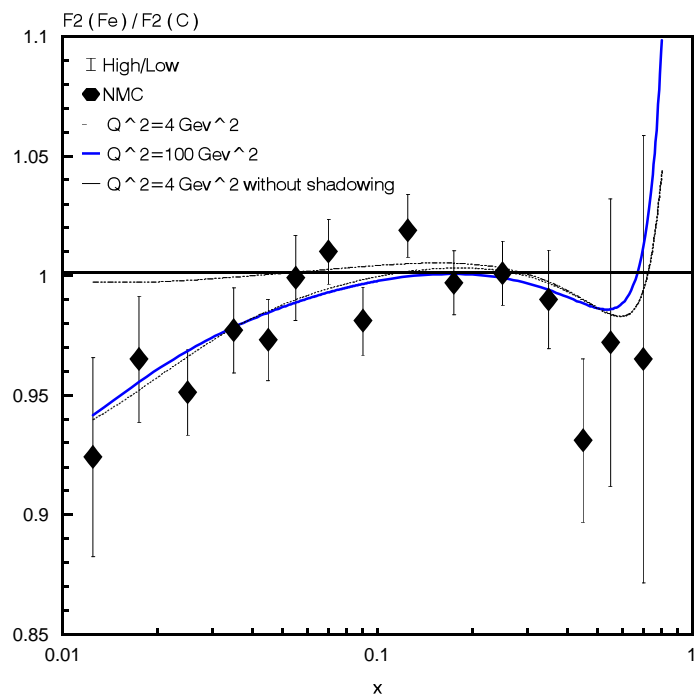c_v &= -0.844 + .451s^{.17}
\end{aligned}
\tag{33}
$$

FIGURES

FIG. 1. F_2^p as a function of x at several Q^2.

FIG. 2. The ratio $R = \frac{F_2^{A_1}(x,Q^2)}{F_2^{A_2}(x,Q^2)}$ for a variety of nuclei at different Q^2. Dashed-dotted line represents the results without the shadowing effects. Others include the shadowing corrections.

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F2 (Ca) / F2 (D)

- High/Low
- NMC
- Q ^ 2 = 4 Gev ^ 2
- Q ^ 2 = 100 Gev ^ 2
- Q ^ 2 = 4 Gev ^ 2 without shadowing

F2 (Ca) / F2 (Li)

- High/Low
- NMC
- Q ^ 2 = 4 Gev ^ 2
- Q ^ 2 = 100 Gev ^ 2
- Q ^ 2 = 4 Gev ^ 2 with

F2 (Ca) / F2 (C)

- High/Low
- NMC
- Q ^ 2 = 4 Gev ^ 2
- Q ^ 2 = 100 Gev ^ 2
- Q ^ 2 = 4 Gev ^ 2 With out shadowing

F2 (He) / F2 (D)

- High/Low
- NMC
- Q ^ 2 = 4 Gev ^ 2
- Q ^ 2 = 100 Gev ^ 2
- Q ^ 2 = 4 Gev ^ 2 with ou

F2 (Fe) / F2 (C)

1.1

⊥ High/Low

● NMC

- Q^2=4 Gev^2

— Q^2=100 Gev^2

— Q^2=4 Gev^2 without shadowing

1.05

1

0.95

0.9

0.85

0.01   0.1   1

x

F2 (C) / F2 (Li)

1.1

1.05

1

0.95

0.9

⊥ High/Low

● NMC

- Q^2=4 Gev^2

— Q^2=100 Gev^2

— Q^2=10 Gev^2 Without s

0.85

0.001   0.01   0.1

x

F2 (Sn) / F2 (C)

1.1

⊥ High/Low

● NMC

- Q^2=4 Gev^2

— Q^2=100 Gev^2

— Q^2=4 Gev^2 Without shadoing

1.05

1

0.95

0.9

0.85

0.8

0.01   0.1   1

x

FIG. 1.

F2 (C) / F2 (D)

⊥ High/Low
● NMC
- Q ^ 2=4 Gev ^ 2
— Q ^ 2=100 Gev ^ 2
— Series 10

F2 (Al) / F2 (C)

⊥ High/Low
● NMC
- Q ^ 2=4 Gev ^ 2
— Q ^ 2=10 Gev ^ 2
— Q ^ 2=4 Gev ^ 2 Without shadowing

F2 (Be) / F2 (C)

⊥ High/Low
● NMC
- Q ^ 2=4 Gev ^ 2
— Q ^ 2=100 Gev ^ 2
— Series 10

F2 (Pb) / F2 (C)

⊥ High
● NMC
- Q ^
— Q ^
— Serie